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Slewing Multimode Flexible Spacecraft with Zero Derivative Robustness Constraints

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Introduction

THE problem of controlling flexible space structures in the presence of modeling uncertainties is an area of active research. For systems equipped with on-off actuators such as reaction jets, the problem is compounded because the command is discontinuous. Techniques for generating on-off command profiles have received much attention recently. One method, known as input shaping, is implemented by convolving a sequence of impulses, an input shaper, with a desired system command to produce a shaped input that is then used to drive the system.[1,2] If the impulse amplitudes are set to specific values and convolved with a step, then the resulting command profile will be a series of pulses realizable with on-off actuators.[3]

The impulse amplitudes and time locations are determined by satisfying a set of constraint equations. If the constraints require only zero residual vibration, then the resulting shaper is called a zero vibration (ZV) shaper. The earliest incarnation of ZV shaping was the technique of posicast control developed by Smith in the 1950s.[2] Singer and Seering developed robust input shaping by also setting the derivative with respect to the frequency of the residual vibration equal to zero.[1] The resulting shaper is called a zero vibration and derivative (ZVD) shaper.

The idea of generating robustness by setting the derivative of the final state equal to zero was used by Banerjee[4] to slew large space-based antennae, by Liu and Wie[5] to produce on-off commands, and by Liu and Singh[6] to generate commands for nonlinear systems. Singhose et al.[3] proposed an alternative robustness technique.

This Note investigates on-off input shapers for multimode flexible spacecraft. Characteristics of the solution space for two-mode systems are discussed and then the robustness to modeling errors are quantified. We show that the robustness to errors in the second mode is highly dependent on the mode ratio.

Multimode On-Off ZVD Input Shaping

The impulses that compose an on-off input shaper are determined by satisfying five types of constraints: amplitude constraints, residual vibration constraints, robustness constraints, rigid-body constraints, and the requirement of time optimality.

A multiswitch bang-bang profile, which can be used with on-off actuators, can be generated by convolving a step input with an input shaper of the form[3]

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 & -2 & \cdots & -2 & 1 \\ 0 & t_2 & t_3 & t_4 & \cdots & t_{n-1} & t_n \end{bmatrix} \tag{1}$$

A_i and t_i are the amplitudes and time locations of the impulses and n is the number of impulses. Equation (1) sets the impulse amplitudes; the impulse times are determined by the remainder of the constraint equations.

The constraint on residual vibration amplitude can be conveniently expressed as the ratio of residual vibration amplitude with shaping to that resulting from a step input. For the kth mode of natural frequency, ω_k, and damping ratio, ζ_k, this percentage vibration is given by[1]

$$V(\omega_k, \zeta_k) = \exp(-\zeta_k \omega_k t_n)\sqrt{[C(\omega_k, \zeta_k)]^2 + [S(\omega_k, \zeta_k)]^2} \tag{2}$$

where

$$C(\omega_k, \zeta_k) = \sum_{i=1}^{n} A_i \exp(\zeta_k \omega_k t_i) \cos\left(\omega_k \sqrt{1-\zeta_k^2}\, t_i\right) \tag{2a}$$

$$S(\omega_k, \zeta_k) = \sum_{i=1}^{n} A_i \exp(\zeta_k \omega_k t_i) \sin\left(\omega_k \sqrt{1-\zeta_k^2}\, t_i\right) \tag{2b}$$

The zero vibration constraints are then k versions of Eq. (2) with V set equal to zero.

In addition to limiting residual vibration amplitude, ZVD shaping requires some amount of robustness to modeling errors by setting the derivative with respect to the frequency of the residual vibration equal to zero.[1] That is,

$$0 = \frac{\mathrm{d}}{\mathrm{d}\omega_k}[V(\omega_k, \zeta_k)] \tag{3}$$

Constraints on the rigid-body motion are also needed. For a system modeled as a series of masses, springs, and dampers, the rigid-body velocity is

$$v_d = \int_0^{t_n} \frac{u(t)}{M}\,\mathrm{d}t \tag{4}$$

where v_d is the desired terminal velocity, $u(t)$ is the input force, and M is the total system mass. For rest-to-rest slewing, v_d equals zero at the end of the slew, $t = t_n$. Integrating Eq. (4) gives a constraint on move distance, x_d:

$$x_d = \int_0^{t_n}\int_0^{t_n} \frac{u(t)}{M}\,\mathrm{d}t\,\mathrm{d}t \tag{5}$$

In a flexible rotary system, the transient deflection may cause a time-varying moment of inertia. These cases may require a more general form of rigid-body constraints.

Because of the transcendental nature of Eqs. (2) and (3), there will be multiple solutions. To make the solution time optimal, the shaper

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must be made as short as possible. Therefore, the time optimality requirement is

$$\min(t_n) \tag{6}$$

Characteristics of Multimode On-Off ZVD Shaping

The time-optimal multimode ZVD shaper is obtained by satisfying the constraints described above. The amplitudes of the shaper impulses are given by Eq. (1). The time locations of the impulses are obtained by satisfying Eqs. (2–5), while using an optimization to minimize t_n.

The solution is a function of the frequencies (ω_i), the damping ratios (ζ_i), the frequency ratios (r_i), the move distance (x_d), and the actuator force-to-mass ratio (FM). All results presented here are for a low frequency of 1 Hz. Furthermore, the discussion is restricted to two-mode undamped systems whose mode ratio is r.

Characteristics of the solution space are presented by varying the move distance or mode ratio. Basing the problem formulation on a three-mass, two-spring model, the force-to-mass ratio is fixed by setting the total mass equal to one and then setting $u_{\max}$ equal to the desired value of FM. The values of the masses and spring constants, k_1 and k_2, are chosen so that the low mode equals 1 Hz and the second mode equals r Hz.

Figure 1 shows the impulse time locations as a function of r when $x_d = 0.5$ and FM $= 1$. As r increases, the slew duration tends to decrease (the time location of the final impulse, t_{11}, is decreasing). The maneuver time decreases rapidly as r increases from 1 to 2, but then it levels off. When the solutions are plotted as a function of move distance, the solution can have various degrees of complexity depending on the mode ratio. To illustrate the most general features, r is set equal to 4.4. Figure 2 shows the impulse time locations when x_d is varied and FM $= 1$. For certain ranges, both the number of impulses and their time locations change rapidly.

In the regions where the solution changes rapidly, finding the time-optimal solution can be difficult, because there are many alternative solutions that are very nearly time optimal. Fortunately, a procedure for verifying the time optimality of numerically obtained solutions has been developed.[7] This procedure was used to verify the solutions shown in Figs. 1 and 2.

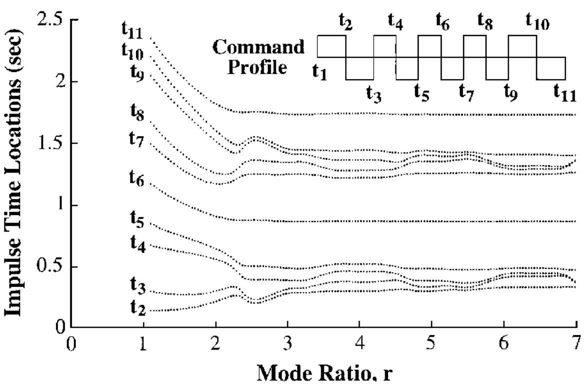

Fig. 1 Impulse times as a function of mode ratio for x_d = 0.5 and FM = 1.

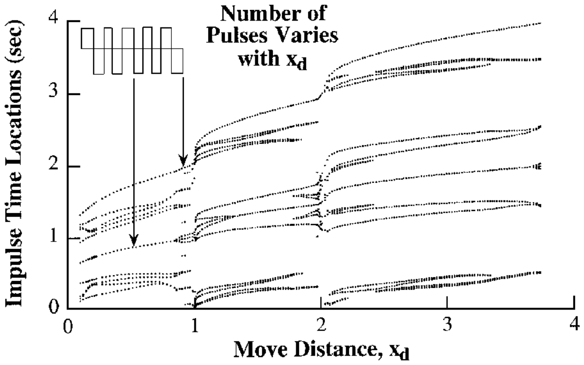

Fig. 2 Impulse times as a function of move distance for r = 4.4 and FM = 1.

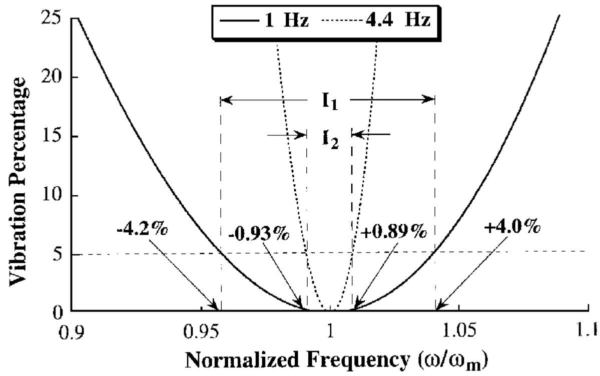

Fig. 3 Calculation of 5% insensitivity.

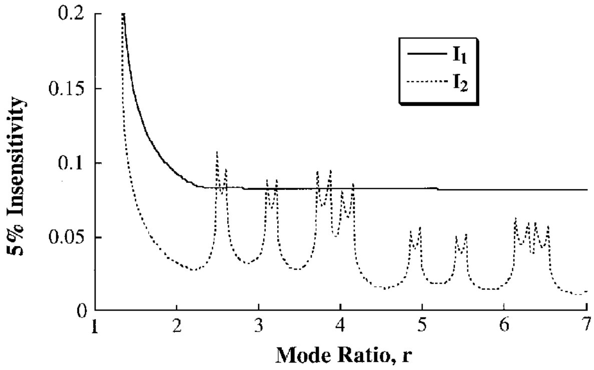

Fig. 4 Insensitivity as a function of mode ratio for x_d = 0.5 and FM = 1.

Evaluation of Robustness

Although the zero derivative constraint attempts to produce robustness to modeling errors, the degree of robustness is not specified, so it must be evaluated quantitatively. For the purposes of this Note, a modeling error refers to an error in estimation of the resonance frequencies of the system. There can, of course, be errors in the estimation of system damping. However, errors in damping generally have less impact on the residual vibration.[1]

An input shaper's robustness to modeling errors can be displayed by a sensitivity curve, a plot of the residual vibration amplitude as a function of frequency. One method of measuring robustness quantitatively is to determine the range of frequencies over which each mode can vary, while the vibration remains below an acceptable level. To obtain a nondimensional measure that is applicable to all frequency values, the frequency ranges are normalized by the modal frequencies. The resulting values give a measure of the allowable percentage change in each mode.

Figure 3 shows two normalized sensitivity curves; one curve is normalized by the 1-Hz mode and the other is normalized by the 4.4-Hz mode. These curves show the residual vibration over a range that is $\pm 10\%$ of each mode. If we assume that 5% residual vibration is acceptable, then the 1-Hz mode can deviate $+4.0$ and -4.2%. This frequency range is called the 5% insensitivity for the low mode, I_1. In this case, $I_1 = 0.082$. The second mode can vary only $+0.89$ and -0.93%. That is, $I_2 = 0.0182$. Although ZVD constraints are enforced at both modes, the robustness is >4 times better for the low mode.

Figure 4 shows both I_1 and I_2 as functions of r when $x_d = 0.5$ and FM $= 1$. These data correspond to the shapers shown in Fig. 1. Although the insensitivity for the low mode is well behaved and remains at a reasonable level for all values of r, the second mode insensitivity varies greatly and is very low for certain mode ratios.

Conclusions

A technique for designing on-off command profiles for multimode flexible spacecraft has been investigated. An attempt to obtain robustness is made by setting the derivative of the residual vibration with respect to the frequencies equal to zero. The complicated nature of the solution space has been illustrated, and the robustness of the method to modeling errors has been quantified. The method has a reasonable level of robustness to errors in the low mode. The robustness to errors in the second mode varies greatly and is poor for large ranges of the system parameters. These results indicate that robustness techniques for multimode systems that are based on limiting the derivative of the final state with respect to system parameters may not be effective for all parameter values.

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Range-Rate Control Algorithms and Space Rendezvous Schemes

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Introduction

SPACECRAFT rendezvous, containing a maneuvering spacecraft and a target spacecraft, consists of two successive phases: long-distance navigation and short-distance homing. In the navigation phase, the orbit parameters as well as the positions of the two spacecraft may differ greatly. The main task of this phase is an impulsive orbit transfer, usually controlled from the ground station. This is an open-loop control. Both orbit determination and impulse generation techniques are applied. The achievable control accuracy of these techniques is rather below that required for the subsequent docking of the two spacecraft. Therefore, the goal of the navigation phase is to bring the maneuvering spacecraft to within a specified vicinity of the target spacecraft (for example, less than 100 km) in the same orbit plane from which the homing phase or terminal rendezvous begins.

In the homing phase, interest is in the motion of the maneuvering spacecraft relative to the target spacecraft. Control of the motion is carried out in such a way that the maneuvering spacecraft moves closer toward the target spacecraft along a stable trajectory and finally stops near it.

Clohessy and Wiltshire[1] developed a control method based on the analytical solution to the linearized differential equations of the relative motion in circular orbits. As the method is open loop, it is very sensitive to the modeling error as well as to the error of control impulse generation. Whereas the method can ensure the end position of the maneuvering spacecraft, it may not be able to ensure a desirable intermediate course of the trajectory in the motion. Another widely used control method is based on the proportional/parallel navigation law.[2] This method is based solely on kinematics and does not take into account the specific dynamics of the motion, which results in degradation of the control performance.

In Refs. 3 and 4, a control method, called the range-rate control algorithm (RRCA), and its modification, that is, the omnidirectional range-rate control algorithm (ODRRCA), have been proposed for terminal rendezvous control. These control algorithms, based on nonlinear dynamical system theory, more clearly display the dynamics of the motion such as the fixed point (equilibrium state) and its stability. They also demonstrate very good performance and implementability.

However, all of these methods are only effective for the target spacecraft in a circular orbit. In the case of an elliptic orbit, the dynamics of the motion becomes more complicated. Instead of a fixed point, there is a limit cycle (periodic motion) in the system. This Note addresses RRCA and ODRRCA, applied to space rendezvous in an elliptic orbit, and suggests three generalized schemes for the terminal rendezvous control.

Equations of Motion and Control Algorithms

The equations of the coplanar rendezvous motion, written in a reference system fixed with the line of sight of the spacecraft, are given as[3]

$$\ddot{D} - D(\dot{v} + \dot{\varphi})^2 + \left(\mu/R_T^3\right)D(1 - 3\sin^2\varphi) = a_S \tag{1}$$

$$D(\ddot{v} + \ddot{\varphi}) + 2\dot{D}(\dot{v} + \dot{\varphi}) - 3\left(\mu/R_T^3\right)D\sin\varphi\cos\varphi = a_\varphi \tag{2}$$

where D is the distance between the two spacecraft, φ is the direction angle or phase angle of the line of sight measured from the local horizontal, v is the true anomaly, R_T is the orbital radius of the target spacecraft, μ is the gravitational coefficient, and a_S and a_φ are forced-acceleration components of the spacecraft. As $\dot{D} \le 0$ in the rendezvous motion, $2\dot{D}\dot{\varphi}$ in Eq. (2) is zero or negative damping, which indicates an instability of the motion. Therefore, the control algorithms, however proposed, should stabilize the motion.

Two control algorithms proposed for the terminal rendezvous are as follows.

RRCA and the Controlled Motion

There is a reference range rate $\dot{D}_r$ in RRCA,[3] defined as

$$\dot{D}_r = \frac{k\dot{v} + k_1\dot{\varphi}}{\dot{v} + \dot{\varphi}}\dot{v}D, \qquad k \in [-0.75, 0], \qquad k_1 > 0$$

where k and k_1 are selectable parameters of the RRCA. The control system will make $\dot{D}$ equal to $\dot{D}_r$ by regulating only the propulsion a_S of the maneuvering spacecraft aligned with the direction of the line of sight (the so-called in-line propulsion) in the form

$$a_S = j(\dot{D}_r - \dot{D}), \qquad j > 0, \qquad a_\varphi = 0$$

The control error ratio $(\dot{D}_r - \dot{D})/|\dot{D}_r|$ is so small[3] that the controlled range rate $\dot{D}$ and, therefore, the distance D can be approximated by

$$\dot{D} = \frac{k\dot{v} + k_1\dot{\varphi}}{\dot{v} + \dot{\varphi}}\dot{v}D \tag{3}$$

The directional motion of φ can be obtained by substituting Eq. (3) into Eq. (2) as follows:

$$\ddot{\varphi} + 2k_1\dot{v}\dot{\varphi} - 1.5\left(\mu/R_T^3\right)\sin 2\varphi = -2k\dot{v}^2 - \ddot{v} \tag{4}$$

ODRRCA and Controlled Motion

ODRRCA consists of two parts as follows.

1) In-line propulsion a_S is the same as for RRCA; thus, the distance motion is given by Eq. (3).

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